NAG Toolbox for MATLAB

f08gp

1 Purpose

f08gp computes selected eigenvalues and, optionally, eigenvectors of a complex n by n Hermitian matrix A in packed storage. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

2 Syntax

```
[ap, m, w, z, jfail, info] = f08gp(jobz, range, uplo, n, ap, vl, vu, il,
iu, abstol)
```

3 Description

The Hermitian matrix A is first reduced to real tridiagonal form, using unitary similarity transformations. The required eigenvalues and eigenvectors are then computed from the tridiagonal matrix; the method used depends upon whether all, or selected, eigenvalues and eigenvectors are required.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Demmel J W and Kahan W 1990 Accurate singular values of bidiagonal matrices *SIAM J. Sci. Statist. Comput.* **11** 873–912

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz – string**

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

2: **range – string**

If **range** = 'A', all eigenvalues will be found.

If **range** = 'V', all eigenvalues in the half-open interval (**vl**, **vu**] will be found.

If **range** = 'I', the **il**th to **iu**th eigenvalues will be found.

Constraint: **range** = 'A', 'V' or 'I'.

3: **uplo – string**

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

4: **n – int32 scalar**

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5: **ap(*) – complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The n by n Hermitian matrix A, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + j(j-1)/2)$ for $i \leq j$;
if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + (2n-j)(j-1)/2)$ for $i \geq j$.

6: **vl – double scalar**
7: **vu – double scalar**

If **range** = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.

If **range** = 'A' or 'I', **vl** and **vu** are not referenced.

Constraint: if **range** = 'V', $\mathbf{vl} < \mathbf{vu}$.

8: **il – int32 scalar**
9: **iu – int32 scalar**

If **range** = 'I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned.

If **range** = 'A' or 'V', **il** and **iu** are not referenced.

Constraints:

if $\mathbf{n} = 0$, $\mathbf{il} = 1$ and $\mathbf{iu} = 0$;
if $\mathbf{n} > 0$, $1 \leq \mathbf{il} \leq \mathbf{iu} \leq \mathbf{n}$.

10: **abstol – double scalar**

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to

$$\mathbf{abstol} + \epsilon \max(|a|, |b|),$$

where ϵ is the ***machine precision***. If **abstol** is less than or equal to zero, then $\epsilon\|T\|_1$ will be used in its place, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues will be computed most accurately when **abstol** is set to twice the underflow threshold $2 \times$ x02am( ), not zero. If this function returns with $\mathbf{info} > 0$, indicating that some eigenvectors did not converge, try setting **abstol** to $2 \times$ x02am( ). See Demmel and Kahan 1990.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

ldz, work, rwork, iwork

5.4 Output Parameters

1: **ap**($*$) **– complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

ap contains the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A.

2: **m – int32 scalar**

The total number of eigenvalues found.

If **range** = 'A', $\mathbf{m} = \mathbf{n}$.

If **range** = 'V', the exact value of **m** is not known in advance, but will satisfy $0 \leq \mathbf{m} \leq \mathbf{n}$.

If **range** = 'I', $\mathbf{m} = \mathbf{iu} - \mathbf{il} + 1$.

3: **w**($*$) **– double array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

If **info** = 0, the selected eigenvalues in ascending order.

4: **z**(**ldz**,$*$) **– complex array**

The first dimension, **ldz**, of the array **z** must satisfy

if **jobz** = 'V', $\mathbf{ldz} \geq \max(1, \mathbf{n})$;
$\mathbf{ldz} \geq 1$ otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{m})$ if **jobz** = 'V', and at least 1 otherwise

If **jobz** = 'V', then if **info** = 0, the first m columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the ith column of Z holding the eigenvector associated with $\mathbf{w}(i)$.

If an eigenvector fails to converge, then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in **jfail**.

If **jobz** = 'E', **z** is not referenced.

Note: you must ensure that at least $\max(1, \mathbf{m})$ columns are supplied in the array **z**; if **range** = 'V', the exact value of **m** is not known in advance and an upper bound must be used.

5: **jfail**($*$) **– int32 array**

Note: the dimension of the array **jfail** must be at least $\max(1, \mathbf{n})$.

If **jobz** = 'V', then if **info** = 0, the first **m** elements of **jfail** are zero.

If **info** > 0, **jfail** contains the indices of the eigenvectors that failed to converge.

If **jobz** = 'E', **jfail** is not referenced.

6: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **range**, 3: **uplo**, 4: **n**, 5: **ap**, 6: **vl**, 7: **vu**, 8: **il**, 9: **iu**, 10: **abstol**, 11: **m**, 12: **w**, 13: **z**, 14: **ldz**, 15: **work**, 16: **rwork**, 17: **iwork**, 18: **jfail**, 19: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, then i eigenvectors failed to converge. Their indices are stored in array **jfail**. Please see **abstol**.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3.

The real analogue of this function is f08gb.

9 Example

```
jobz = 'Vectors';
range = 'Values in range';
uplo = 'U';
n = int32(4);
ap = [complex(1, +0);
     complex(2, -1);
     complex(2, +0);
     complex(3, -1);
     complex(3, -2);
     complex(3, +0);
     complex(4, -1);
     complex(4, -2);
     complex(4, -3);
     complex(4, +0)];
vl = -2;
vu = 2;
il = int32(-1208245600);
iu = int32(121387641);
abstol = 0;
[apOut, m, w, z, jfail, info] = f08gp(jobz, range, uplo, n, ap, vl, vu,
il, iu, abstol)
```

```
apOut =
  -0.2187
   1.0422
  -0.3942
   0.4448 + 0.4277i
  -3.4564
   6.6129
   0.3367 + 0.0008i
   0.3567 - 0.0783i
  -7.8740
   4.0000
m =
           2
w =
```

```
   -0.6886
    1.1412
         0
         0
z =
  -0.3975 + 0.5105i  -0.3746 - 0.2414i
   0.3953 - 0.3238i   0.2895 - 0.4917i
  -0.4309 + 0.0383i   0.3768 + 0.3994i
   0.3648            -0.4175
jfail =
           0
           0
           0
           0
info =
           0
```